Helium in chirped laser fields as a time-asymmetric atomic switch

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Tuning the laser parameters exceptional points in the spectrum of the dressed laser helium atom are obtained. The weak linearly polarized laser couples the ground state and the doubly excited P-states of helium. We show here that for specific chirped laser pulses that encircle an exceptional point one can get the time-asymmetric phenomenon, where for a negative chirped laser pulse the ground state is transformed into the doubly excited auto-ionization state, while for a positive chirped laser pulse the resonance state is not populated and the neutral helium atoms remains in the ground state as the laser pulse is turned off. Moreover, we show that the results are very sensitive to the closed contour we choose. This time-asymmetric state exchange phenomenon can be considered as a time-asymmetric atomic switch. The optimal time-asymmetric switch is obtained when the closed loop that encircles the exceptional point is large, while for the smallest loops, the time-asymmetric phenomenon does not take place. A systematic way for studying the effect of the chosen closed contour that encircles the exceptional point on the time-asymmetric phenomenon is proposed. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4885136]

I. INTRODUCTION

Let us begin the introduction by describing the development of the different fields in optics and molecular physics where non-Hermitian degeneracies that are known in the literature as exceptional points1,2 in the spectrum of the open system are obtained and control the dynamics. For a long time exceptional points were considered as mathematical objects only (as for example in perturbation theory where the radius of convergence is associated with a branch point in the complex energy plane as the perturbation strength parameter is analytically continued into the complex plane3,4). Another example, the exceptional points are obtained in the complex scaled variational calculations due to the use of finite number basis functions which can be as large as one wishes.5 Only later it was found that there are physical phenomena where exceptional points appear naturally. The phenomena which result from the exceptional points were theoretically studied in atomic and molecular physics,6–10 in electron scattering from hydrogen molecules,11 superconductors,12 quantum phase transitions in a system of interacting bosons,13 electric field oscillations in microwave cavities,14 in PT-symmetric waveguides,15,16 and in unstable laser resonators.17 Exceptional points can also arise in diffraction from absorbing gratings18 and in crystal optics.19

In molecular physics (such as in electron scattering from molecules), the exceptional points (so called branch points) were not measured directly but were shown to be important identities in the use of non-Hermitian scattering theory to explain sharp peaks in cross-sections due to temporary trapping of electrons by the hydrogen molecule in spite of the fact that the lifetime of the molecular negative ions (H−) is extremely small (about a femtosecond).11 More recently it has been shown that in the photo-dissociation of diatomic molecules by lasers exceptional points in the spectrum of the Floquet Hamiltonian are obtained due to the coalescence of two photo-pre-dissociation states (so called resonances),20 or that complex degeneracies in nuclear potential energy surfaces have a strong impact on resonant Auger spectra and electron yields in strong laser intensities of hundreds of PW/cm2.10 Moreover, it was shown that by varying the laser parameters (where time is used as instantaneous parameter and not a dynamical parameter as it should) in a closed loop that encircles the exceptional point one gets the adiabatic state exchange phenomenon.20 That is, the initial state |0⟩ turns to be |1⟩ while the initial state |1⟩ is exchanged to a final state |0⟩. However, as the laser parameters are varied in time in a closed loop that encircles the exceptional point the situation is quite different.21,22 If for a negative (positive) chirped laser experiment an exchange from one field free-state to another occurs, in the positive (negative) chirped laser experiments the final state will be as the initial one.23,24 To the best of our knowledge, time-asymmetric phenomenon has not been studied before for the case where the resonances are induced by the atomic photo-ionization processes rather than the molecular photo-dissociation processes. In this work, we study the possibility to obtain the time-asymmetry when exceptional points result from the coalescence of an auto-ionization resonance (due to the electronic correlation) with

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a photo-induced resonance state. Before describing in detail the way that exceptional points are obtained for the helium atom which interacts with a linear polarized laser field we will briefly describe our computational approach.

One of the possibilities to describe extensive motion of driven and excited electrons in atoms and molecules is using extensive Gaussian basis sets. For the present problem, dealing with autoionization, it is no less important that the basis sets comply well with the complex scaling method. It has been pointed out that the standard basis sets are generally not suitable for complex scaling calculations of atoms, as particularly manifested in large artificial widths of the bound states.

A Floquet Hamiltonian

The exceptional points that are in focus here are solutions of the Floquet Hamiltonian, where the field-free states of helium atom are coupled by a linearly polarized laser in the dipole approximation. The complex scaled Floquet Hamiltonian is defined as

\[ \hat{H}_\theta = \hat{H}_\theta(t) - i \hbar \frac{\partial}{\partial t}, \]

where \( \hat{H}_\theta(t) \) is given by

\[ \hat{H}_\theta(t) = \hat{H}_\theta^{(0)} + e e^{i \omega t} (\zeta_1 + \zeta_2) \varepsilon_0 \sin \omega t, \]

where \( \hat{H}_\theta^{(0)} \) is a complex scaled field-free Hamiltonian, \( e \) is the elementary charge, \( \zeta_1, \zeta_2 \) represent electronic coordinates, \( \omega \) is the leading laser frequency, and \( \varepsilon_0(t) \) represents the electric field, which is related to the field strength \( I_0 \) via the relation

\[ \varepsilon_0 = \sqrt{\frac{2I_0}{c_0 e^2}}, \]

where \( c_0 \) is the speed of light and \( \varepsilon_0 \) is the vacuum permittivity.
B. Conjugate pairs of exceptional points obtained within a framework of two-Floquet-level approximation

The following discussion should give us a basic insight to where and why exceptional points between helium quasi-energy states happen to occur. Let us assume a Floquet Hamiltonian, where only the helium ground state (with energy $E_{GS}$) and a particular helium doubly excited autoionization resonance (with complex energy $E_{ES} - i\Gamma_{ES}/2$) are coupled by the field $\varepsilon_0$ with the frequency $\omega$ through the transition dipole moment $\mu$, which is complex. For a near-resonance frequency, it makes sense to assume the one-electron approximation, where the Floquet Hamiltonian boils down to

$$\mathcal{H} = \begin{bmatrix} E_{ES} - i\Gamma_{ES}/2 & i\varepsilon_0\mu/2 \\ i\varepsilon_0\mu/2 & E_{GS} + \hbar\omega \end{bmatrix}.$$  \tag{3}

Note that this non-Hermitian Floquet Hamiltonian implies autoionization, which would manifest itself as a decrease of the total Cartesian norm during dynamical simulations of the photoexcitation.

The two-level non-Hermitian Hamiltonian Eq. (3) is equivalent to the “minimal model” of doubly intersecting complex energy surfaces (DICES), which represent a complex analogy to the famous conical intersection in Hermitian dynamics. Yet, this particular form of Hamiltonian leading to DICES appeared even earlier in the context of vibronic coupling of short-lived electronic states. The solutions of this matrix for different field-strength and frequency include two exceptional points. At each exceptional point, two complex quasi-energies are degenerated (i.e., coalesce). Since the associated two resonance wavefunctions are bi-orthogonal (using c-product rather than scalar Dirac product), it implies that at the exceptional point where the two resonance wavefunctions coalesce a self-orthogonal state is obtained (see Chap. 9 in Ref. 4) and the Dirac norm of the function associated with the exceptional point is infinitely large. The exceptional points are found for the field strengths and frequencies $[\varepsilon_0^{(EP)}, \omega^{(EP)}]$ and $[\varepsilon_0^{(EP)}, \omega^{(EP)}]$ given by

$$\varepsilon_0^{(EP)} = -\varepsilon_0^{(EP)} = \frac{\Gamma_{ES}}{2|m\mu|}, \quad \omega^{(EP)} = \omega^{(EP)} = \frac{E_{ES} - E_{GS}}{\hbar - \hbar\omega} \pm \Delta_{E}.$$ \tag{4}

The solutions of the reduced Floquet matrix in Eq. (3) are given by

$$E_{\pm} = E^{(EP)} + \frac{\hbar(\omega - \omega^{(EP)})}{2} \pm \Delta_{E},$$ \tag{5}

where

$$E^{(EP)} = E_{ES} - \frac{\Gamma_{ES}^2}{4} \left( i + \frac{\mu}{4|m\mu|} \right).$$ \tag{6}

The eigenvectors (not normalized) are defined as

$$c_{\pm} = \begin{bmatrix} E_{ES} - i\Gamma_{ES}/2 - E_{GS} - \hbar\omega \pm \Delta_{E} \\ i\varepsilon_0\mu/2 \end{bmatrix}.$$ \tag{7}

The energy split $\Delta_{E}$ can be expressed in two ways, where either one or the second exceptional point is taken as the reference:

$$\Delta_{E} = \sqrt{a(\omega - \omega^{(EP1)})^2 + \frac{\hbar^2}{4}(\omega - \omega^{(EP1)})^2 + b_1(\varepsilon_0 - \varepsilon_0^{(EP1)})^2 + \frac{\mu^2}{4}(\varepsilon_0 - \varepsilon_0^{(EP1)})^2} = \sqrt{a(\omega - \omega^{(EP2)})^2 + \frac{\hbar^2}{4}(\omega - \omega^{(EP2)})^2 + b_2(\varepsilon_0 - \varepsilon_0^{(EP2)})^2 + \frac{\mu^2}{4}(\varepsilon_0 - \varepsilon_0^{(EP2)})^2}.$$ \tag{8}

The parameters $a$, $b$, and $b_2$ of the exceptional points are given by

$$a = \frac{\hbar\Gamma_{ES}}{4} \left( i - \frac{\mu}{4|m\mu|} \right), \quad b_1 = -b_2 = \frac{\Gamma_{ES}^{2\mu}}{4|m\mu|}.$$ \tag{9}

Note that our approach is in harmony with the Taylor series expansion approach taken in Ref. 30. Note that when we are sufficiently close to one of the exceptional points, we get an expression that has been used before in Refs. 21, 23, and 24, provided of course that $a$ and $b$ are nonzero. Figures 1(a) and 1(b) display the real and imaginary parts of the energy split $\Delta_{E}$ for the coupled GS and resonance $1P^0(a)$.

The two-state estimates for the laser parameters at exceptional points $\hbar\omega^{(EP)}, \varepsilon_0^{(EP)}$, and the maximum frequency dispersion $|\omega - \omega^{(EP)}|$, where the energy split near the exceptional points is governed by the square root of frequency, are given in Table I. The resonance energies and transition dipole moments for the calculation of the branch points are obtained using the complex scaling and ExTG5G exponential tempered Gaussian basis set. The quantities (4) were calculated for different values of the complex scaling parameter $\theta$, while Table I shows only the best variational values, where $|dReAl/d\theta|, |dImAl/d\theta|$ have minima. The number of reported digits is given by an error estimate based on the formula $\Delta_{A} = 0.25\pi dAl/d\theta$, which is applied separately for the real and imaginary parts of the quantity $A$ in a way that is analogous to our previous calculations of the Einstein coefficients in Ref. 26.

The doubly excited resonances in the $1P^0(c)$ series have such narrow widths that they fall below the precision of our calculations, which is $10^{-6} E_{0}$. Nevertheless, the real parts of the complex energies and transition dipole moments are obtained within a reasonable accuracy even for this series. When these values are substituted to Eq. (4), we obtain that the frequencies of the exceptional points are approximately given by the resonance frequencies, while the laser intensities for the exceptional points are smaller than $10^{-3}$ TW/cm$^2$.
C. The one-photon exceptional points in a full-dimensional approach

The two-Floquet-level approximation corresponds to using a very small basis set, which should normally include bound states, doubly excited resonances, the rotated continuum for several ionization thresholds, and doubly ionized states. Not surprisingly, in some cases, the complex quasi-energy $E_{±}$ acquires a positive definite imaginary part when calculated within the two-Floquet-level model. Such a situation occurs whenever the imaginary part of the quasi-energy split exceeds one half of the resonance widths, i.e., $\text{Im} \Delta E > \Gamma_{E±}/4$. For example, in the case of the $2^1P^0(a)$ resonance, this occurs for the vast regions where $\text{Im} \Delta E > 0.343 \text{ mE}_{h}$, see Fig. 1(b). Not least for this reason it is essential that the predictions made for the two-Floquet-level approach are approved in a converged calculation using the full Floquet matrix.

A general iterative procedure is used to search for a complex degeneracy of quasi-energies in the frequency and field-strength plane, where the initial approximate values $\omega_{±}^{(EP)}$, $\epsilon_{±}^{(EP)}$ are given by the two-Floquet-level approxima-

tion, Eq. (4). Approximately evaluated Cartesian norms grow by up to five orders of magnitude when approaching the degeneracy making us sure of indeed approaching an exceptional point. (Note that at the exceptional point the two $c$-normalized states coalesce being self-orthogonal, therefore their scalar product diverges.4) The Floquet problem is calculated based on the $(t, I)$-method,31-34 where all relevant numerical details have been already discussed in Ref. 26.

The two strongly coupled states give rise to a pair of quasi-energies $E_{±}$ that follow the formula given by

$$E_{±} \approx \bar{E} \pm \Delta E,$$

where the energy split is expanded as

$$\Delta E = \left[ a(\omega - \omega^{(EP)}_±) + b(\epsilon_{±} - \epsilon_{±}^{(EP)}) + c_1(\omega - \omega^{(EP)}_±)(\epsilon_{±} - \epsilon_{±}^{(EP)}) + c_2(\omega - \omega^{(EP)}_±)(\epsilon_{±} - \epsilon_{±}^{(EP)})^{2/3/2} \right],$$

TABLE I. Estimates of exceptional points based on the two-Floquet-state approximation (Eq. (4)). The complex energies and transition dipole moments that are substituted to Eq. (4) are based on complex scaled Full-CI calculations using the ExTG5G basis set, where the variational values with respect to the complex scaling parameter $\theta$ are considered.

| Excited state | $\Gamma_{ES}(E_{±})$ | $\hbar \omega^{(EP)}(E_{±})$ | $\epsilon_{±}^{(EP)}$(TW/cm²) | $\theta|\omega - \omega^{(EP)}(E_{±})|$ |
|---------------|-----------------|-----------------|-----------------|-----------------|
| $1^S \rightarrow 2^1P^0(a)$ | 0.692991 | 0.001372 | 2.21027 | 12.7 | 0.00146 |
| $1^S \rightarrow 3^1P^0(a)$ | 0.56406 | 0.0003 | 2.33939 | 3.9 | 0.000322 |
| $1^S \rightarrow 4^1P^0(a)$ | 0.5343528 | 0.000128 | 2.369127 | 1.76 | 0.0001376 |
| $1^S \rightarrow 5^1P^0(a)$ | 0.521499 | $6.6 \times 10^{-5}$ | 2.381992 | 0.91 | $7.1 \times 10^{-5}$ |
| $1^S \rightarrow 3^1P^0(b)$ | 0.5970734 | $3.788 \times 10^{-6}$ | 2.3064313 | 0.01697 | $3.894 \times 10^{-6}$ |
| $1^S \rightarrow 4^1P^0(b)$ | 0.5464832 | $1.88 \times 10^{-6}$ | 2.3570217 | 0.0148 | $1.97 \times 10^{-6}$ |
| $1^S \rightarrow 5^1P^0(b)$ | 0.5272927 | $8 \times 10^{-7}$ | 2.3762124 | 0.00683 | $9.18 \times 10^{-7}$ |
| $1^S \rightarrow 6^1P^0(b)$ | 0.517934 | $4.6 \times 10^{-7}$ | 2.3855706 | 0.003 | $5 \times 10^{-7}$ |
| $1^S \rightarrow 4^1P^0(c)$ | 0.5470713 | $<10^{-7}$ | 2.3564399 | $<10^{-7}$ | $<10^{-7}$ |
| $1^S \rightarrow 5^1P^0(c)$ | 0.527604 | $<10^{-7}$ | 2.3759012 | $<10^{-7}$ | $<10^{-7}$ |
| $1^S \rightarrow 6^1P^0(c)$ | 0.518111 | $<10^{-7}$ | 2.3853941 | $<10^{-7}$ | $<10^{-7}$ |
and the energy average is given by
\[
\bar{E} = E^{(EP)} + \alpha_0(\omega - \omega_0^{(EP)}) + \beta_0(\epsilon_0 - \epsilon_0^{(EP)}) + \gamma_0(\omega - \omega_0^{(EP)})^2 + \beta_0(\epsilon_0 - \epsilon_0^{(EP)})^2 + \gamma_0(\omega - \omega_0^{(EP)})_0(\epsilon_0 - \epsilon_0^{(EP)})^2.
\]
(12)
The coefficients in these formulas have been calculated by examining the higher-order derivatives \(\partial^{l+1} / \partial^{l} \epsilon_l^{(EP)}\) of the complex function \((E_\nu - E_\gamma)^{2}\) in the exceptional point and its close vicinity, where the derivatives have been obtained numerically using the finite differences. It has been found that only a limited number of the higher-order derivatives have non-zero values, therefore only up to the six and seven terms in the expansions are needed, respectively.

D. Numerical details of the calculation of exceptional points with the exponentially tempered Gaussian basis set

In the following, we give an account on a convergence of our calculations with the used basis sets and complex scaling parameter for the case of the one-photon transition \(1S^\pi \rightarrow 2I^Pn(l)\). The reported complex degeneracies \((\Delta \epsilon_\gamma)\) have been calculated up to the precision of 0.01 \(\mu E_\hbar\). Other parameters of the exceptional point \((\omega_0^{(EP)}, \epsilon_0^{(EP)}, I_0^{(EP)})\), \(a, b, a_2, b_2, c_1, c_2\) have been obtained with the accuracies dependent on the limited precision of the complex degeneracy and correspond with the number of digits given in the respective tables. Table II shows the calculated complex degenerate energy \(E^{(EP)}\) and parameters \(\omega^{(EP)}, \epsilon_0^{(EP)}\) of the exceptional point for various sizes of the basis set of the field-free states for one value of the complex scaling parameter \(\theta = 0.2\). We have reduced the number of interacting states systematically according to criteria proposed in Ref. 26, namely: (i) the number of partial waves in the one-electron basis sets used for Full-CI calculated field-free states is limited by \(l \leq l_{\text{max}}\); (ii) the maximum rotational number is limited by \(L \leq L_{\text{max}}\); and (iii) the highest excitation of the lower lying electron is limited by \(n \leq n_{\text{max}}\). The number of interacting states has also been reduced by excluding fast decaying states of rotated continua that live shorter than 8 attoseconds (i.e., the states for which \(\text{Im}E < -3 \epsilon_0^\gamma\)). The first set of calculations in Table II shows that using \(l_{\text{max}} = 2\) is sufficient to get well converged results. The second set of calculations compares two different Gaussian basis sets, ExTG5G and ExTG7F, which differ in number of partial waves \((l_{\text{max}} = 4\) and 3, respectively) and excitation numbers for the highest Rydberg states \((N_{\text{max}} = 5\) and 7, respectively). We assume that the different number of partial waves is crucial here due to the low excitations of the two most involved quasi-energy states, concluding that errors due to the number of partial waves are rather low. The last set of the calculations in Table II shows that an excellent convergence is obtained when the number of interacting states is truncated by using the lower electron excitation only up to \(n_{\text{max}} = 4\), which nearly corresponds to the highest excitation available for the ExTG5G basis set, therefore this step effectively means excluding doubly ionized states. A benchmark Full-CI calculation including all states obtained for the ExTG5G basis set is given in the last line of Table II.

Having determined the basis set of interacting field-free states, we check a sensitivity of the results to the complex scaling parameter. The first set of calculations in Table III represents the \(\theta\)-trajectories for the “converged basis set” defined such that \(l_{\text{max}} = 4\), \(L_{\text{max}} = 2\), and \(n_{\text{max}} = 4\). The calculation shows a stabilization for \(\theta \approx 0.45\). The complex quasi-energies show a dispersion of few \(\mu E_\hbar\) within the tested interval of \(\theta\), which is consistent with the previous results for Feshbach resonances and quasi-energies. However, for all tabulated quantities the dispersion due to the change of \(\theta\) exceeds the error due to the highest excitation number \(n_{\text{max}}\) for \(n_{\text{max}} \geq 2\), compare Table II. It is therefore reasonable to define the “minimum basis set” of the interacting states such that \(l_{\text{max}} = 4, L_{\text{max}} = 2, n_{\text{max}} = 2\) and later check convergence of any results of dynamical calculations for different values of the complex scaling parameter and both “converged” and “minimum basis sets.” The second set of results in Table III

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TABLE III. Dependence of the parameters \( \xi^{(EP)}_0 \) and \( \alpha^{(EP)} \) and the respective complex energies \( \varepsilon^{(EP)} \) of the exceptional point on the complex scaling parameter \( \theta \) for the \( X^1S \rightarrow 2^1P(\alpha) \) transition. The first and second sets of calculations pertain to the “minimum” \( (l_{\text{max}} = 4, L_{\text{max}} = 2, n_{\text{max}} = 2) \), and “converged” \( (l_{\text{max}} = 4, L_{\text{max}} = 2, n_{\text{max}} = 4) \) basis sets of the interacting field-free states.

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<td>177</td>
<td>12.2771221426</td>
<td>2.2102657181</td>
<td>0.69308615</td>
<td>3.5139371 \times 10^{-4}</td>
</tr>
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The \( c_1 \) and \( c_2 \) coefficients show a relatively strong dependence on the complex scaling parameter \( \theta \), as illustrated for the case of the basis set \( l_{\text{max}} = 4, L_{\text{max}} = 2, \) and \( n_{\text{max}} = 4 \) in Table V. A possible influence of the discrepancies of the quadratic and cubic coefficients \( c_1 \) and \( c_2 \) on the choice of optimal pulses and prediction of dynamical phenomena needs to be taken into account. Therefore, calculations will be repeated for both the minimal and middle basis sets and at least two different values of the complex scaling parameter.

### III. DYNAMICAL ASYMMETRIC ATOMIC SWITCH

#### A. Outline of the problem

While examining the coupled potential energy surfaces along a closed curve that encircles the physical exceptional point in the positive half-plane of laser strength, we observe...
that the complex Floquet energies $E_+$ and $E_-$ adiabatically flip at each circle. This flip is demonstrated by a discontinuity-like behavior of $\text{Im} \Delta E$ that changes its sign while $\text{Re} \Delta E = 0$, see Fig. 1. This “discontinuity” appears in our 2D plot due to the multi-valued functional behavior of $\Delta E$ (the difference between the two almost degenerate complex resonance energies) as the two laser parameters are varied. Orbiting along such a closed curve can be practically realized by a chirped pulse. Such an adiabatic transition would be possible if dynamical non-adiabatic corrections approached to zero. In non-Hermitian quantum mechanics, the non-adiabatic corrections do not always approach to zero as pulse lengths approach to infinity, due to non-zero width of the coupled quasi-energy resonance states.

A previous study, Ref. 24, dealt with the case, where the imaginary part of the energy split $\text{Im} \Delta E[\omega(t), \varepsilon(t)]$ had a unique sign all over the region near the exceptional point. The study has suggested that the non-adiabatic corrections were proportional to

$$\Psi_{\pm}^{NA} \propto \frac{1}{\tau} \exp \left\{ \mp \int_0^\tau \text{Im} \Delta E[\omega(t), \varepsilon(t)] dt \right\}, \quad (13)$$

where $\tau$ was the pulse length and the “±” signs denoted the particular Floquet state. In the limit of slow evolution, $\Psi_{\pm}^{NA}$ limits in zero for one state, but in infinity for the other state, dependent on the sign of the imaginary split of energies. This implies, in words of Ref. 24, that “only one state evolves adiabatically while the other state behaves nonadiabatically. . . . For the other states the adiabatic solution is not valid even approximately, making the adiabatic flip . . . impossible.”

The situation becomes more complicated when the energy split $\text{Im} \Delta E[\omega(t), \varepsilon(t)]$ does not have a unique sign all over the region near the exceptional point. For example at a quick look on Fig. 1(b) it is clear that in the case of the studied atomic switch, the sign of $\text{Im} \Delta E[\omega(t), \varepsilon(t)]$ is not unique along any contour, which encircles the exceptional point. Yet, as we will show here it is possible to get a time asymmetric switch like behavior which is heavily dependent on the contour we chose in the two laser parameters space.

B. Contours of the real part of the energy split between the coupled Floquet states

The choice of the contours encircling the exceptional point is not arbitrary due to the fact that the imaginary part of the complex energy split assumes different signs on the $\omega - \varepsilon$ plane. As a reasonable starting point, we choose the contours of the real part of the energy split, where $\text{Re} \Delta E = P = \text{const}$. We also demand that the time-points are distributed evenly along the contours. The energy split $\Delta E$ is approximated by the analytical formula defined in Eqs. (10)–(12) in order to simplify the evaluation of the contour points, such that

$$P = \text{Re} \left[ a(\omega - \omega^{(EP)}) + b(\varepsilon - \varepsilon^{(EP)}) + a_2(\omega - \omega^{(EP)})^2 
+ b_2(\varepsilon - \varepsilon^{(EP)})^2
+ c_1(\omega - \omega^{(EP)})^2(\varepsilon - \varepsilon^{(EP)})^2
+ c_2(\omega - \omega^{(EP)})(\varepsilon - \varepsilon^{(EP)})^2 \right]^{1/2} = \text{const.} \quad (14)$$

on the contours. Note that the numerical calculation of the parameters $a, b, c_1, c_2$, requires evaluations of the Floquet problem for seven points in the $\omega - \varepsilon$ plane only. An algorithm for a calculation of the contours/chirped-pulses defined by Eq. (14) is described in the Appendix—obviously, only the halves of the contours that pertain to the positive half-plane of the laser strength $\varepsilon$ are calculated. Typical contours are demonstrated in Fig. 2 for the one-photon transition $^1S_0 \rightarrow 2^1P(a)$.

The definition of the contours is based on the analytical approximation to the energy split $\Delta E$, Eq. (11). This formula is an approximation even in a close vicinity to the exceptional point. Therefore, its validity is now to be examined for different values of $P$. Figure 3 illustrates the difference of the energy splits $(E_+ - E_-) - \Delta E$, where $(E_+ - E_-)$ is obtained in a full-dimensional calculation, while $\Delta E$ is the analytical approximation. The analytical prediction clearly diverges (with the power of 3.2) from the actual full-dimensional results starting from $P = 0.3 \, mE_0$. This fact combines well with understanding $P$ as an effective radius of the contours. It seems that for contours that are very close to the exceptional points, an almost constant and a relatively large error of $10^{-7} \, E_0$ is obtained. The most probable reason is a

<table>
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<tr>
<th>$\theta$</th>
<th>$\text{Re} \Delta E$</th>
<th>$\text{Im} \Delta E$</th>
</tr>
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<td>0.25</td>
<td>1.52084 $\times 10^{-4}$</td>
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</tr>
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<td>1.54502 $\times 10^{-4}$</td>
<td>$-1.061122 \times 10^{-3}$</td>
</tr>
<tr>
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<td>1.50586 $\times 10^{-4}$</td>
<td>$-1.035247 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.45</td>
<td>1.28368 $\times 10^{-4}$</td>
<td>$-1.051951 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.50</td>
<td>1.25561 $\times 10^{-4}$</td>
<td>$-1.056956 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.55</td>
<td>0.78642 $\times 10^{-4}$</td>
<td>$-1.079080 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
finite numerical precision of the parameters of the exceptional point $\omega^{(EP)}$, $\varepsilon_0^{(EP)}$, $a$, $b$, $a_2$, $b_2$, $c_1$, $c_2$ that limits the precision of the analytical predictions and makes the error larger than expected.

![FIG. 3. A difference of the actual energy split $E_+ - E_-$ obtained in a multidimensional Floquet calculation and the energy split $\Delta \bar{E}$ obtained from the analytical expression Eq. (11), which holds exactly only in the vicinity of the exceptional point. The two nearly degenerate Floquet states are dominated by the field-coupled $X\,^1\!S'$ and $2\,^1\!P^0(a)$ states of helium. The error varies along the contours, therefore we provide the average error (interconnected dots), while the minimum and maximum errors are demonstrated by the gray region. As expected the deviation of the energy splitting from the analytical expression is increased as the “radius” of contour encircles the exceptional point becomes larger, i.e., $P$ gets larger. However, the deviation from the analytical formula does not change monotonically with $P$. The results refer to calculations using the “converged basis set” defined in the text and $\theta = 0.45$.](image)

C. Excitation and ionization of driven helium with near resonance chirped pulses

The contours of $P \in \{0.1, 1, 5\}$ $mE_h$ in the $\omega - \varepsilon_0$ plane define chirped pulses that are illustrated in Fig. 4. As $P$ becomes large, the chirped pulse becomes more-or-less symmetric in time, approximately given by $I_0(t) \approx \max I_0[\sin^2 \omega t/\pi t, \omega(t) \approx E_{ES} - E_{GS} + 0.5 (\max |\omega(t)| - \min |\omega(t)|)\cos \omega t/\pi t]$, see Figs. 4(a) and 4(b). Figure 4(c) illustrates the large span of the peak intensities of the chirped pulses based on the chosen contours of $P$ ranging from about 20 to 2500 $TW/cm^2$. Figure 4(d) shows the span of the leading frequency (wavelength) in the chirps reaching up to 20.5 nm $< \lambda < 20.7$ nm in the strongest pulses.

The helium atom which is exposed to the chirped pulse evolves according to the time-dependent Schrödinger equation, where the Hamiltonian is time-dependent, defined as

$$\hat{H}_P(t) = \hat{H}_0^{(0)} + e^{i\theta} (z_1 + z_2) \varepsilon_0(t; P) \sin |\omega(t); P|\tau \right).$$

The chirped pulses of low intensities such as $P = 0.1 \ mE_h$ (Figs. 5(a)–5(b)) affect mainly the populations of the two strongly coupled states, $X\,^1\!S'$ and $2\,^1\!P^0(a)$. It is straightforward to define relative populations of the two coupled states such that

$$\frac{|c_n|^2}{|c_{GS}|^2 + |c_{ES}|^2},$$

where $c_n$ stands either for $c_{GS}$ of $c_{ES}$, $c_{GS}$ and $c_{ES}$ are projections of the dynamical electronic wavefunction $\phi_{GS}^{(0)}(r; t)$ onto the respective non-Hermitian field-free basis functions. As the excited resonance state $2\,^1\!P^0(a)$ becomes populated, the system gets ionized with the rate approaching the Floquet resonance width $\Gamma = 301.1 \ cm^{-1}$ (i.e., 90% ionization within 40 fs). The resonance ionization decay is typical for the low intensities such as 14 $TW/cm^2$ (Figs. 5(a) and 5(b)), where the 90% ionization is reached within 68 fs, corresponding to the energy width of about 200 $cm^{-1}$. As the intensities become larger, population of discretized continuum becomes non-negligible, therefore the decay of the sum of populations of the two coupled states $|c_{GS}|^2 + |c_{ES}|^2$ is predominantly due to the direct ionization process. This is demonstrated in the increased ionization decay rates for the chirped pulses with the peak intensity of 100 $TW/cm^2$ (Figs. 5(c) and 5(d)), where 90% ionization is reached at different short times of 31 fs and 19 fs (corresponding to energy widths of 410 $cm^{-1}$ and 700 $cm^{-1}$) for the negative and positive pulses, respectively. For the very intense chirped pulse of 2.5 $PW/cm^2$ (Figs. 5(c) and 5(d)), 90% ionization is reached within 7 fs and 4 fs (corresponding to energy widths of 1800 $cm^{-1}$ and 4000 $cm^{-1}$) for the negative and positive pulses, respectively.

D. Numerical method applied to propagation

We have used a propagation method, which has been proposed for pulses of thousands optical cycles in Ref. 26. Below we present its improved and generalized version for the chirped pulses. The pulse is approximated by a stair function composed of $N$ stairs, where each stair has a constant field intensity $I_0^{(j)}$ and frequency $\omega_j$, therefore a propagation of the
wavefunction along each stair is defined by the corresponding Floquet quasienergy states such that

\[ u_{k,k'}^{(j)} = \sum_{k''} e^{i\omega_{j} t} e^{i\Delta_{j}(t)} c_{k,k''}^{0} e^{-i\omega_{j} t}, \]

where \( u_{k,k'}^{(j)} \) is the evolution operator for the \( j \)-th stair in the basis set of the complex scaled field-free states. \( e_{k,k'}^{0} \) are the quasienergies calculated for the \( j \)-th stair and \( c_{k,k'}^{0} \) and \( c_{k,k'}^{0(j)} \) are defined as

\[ e_{k,k'}^{0} = \sum_{n} c_{k,n,k'0} e^{i\omega_{j} t}, \]

\[ c_{k,k'}^{0(j)} = \sum_{n} c_{k,n,k'0} e^{i\omega_{j} t}, \]

where \( c_{kn,k'0} \) and \( c_{kn,k'0(j)} \) are the coefficients of the right and left Floquet vectors calculated for the \( j \)-th stair in the basis set of complex scaled dressed states. All stairs have approximately the same lengths, \( \Delta_{j}(t) \), while we require that each stair length is given by the integer multiplier of the half of the corresponding optical cycle such that

\[ \Delta_{j}(t) = m_{j} \pi/\omega_{j}. \]

With this measure, the oscillations of the electric field \( \sin \omega_{j}(t + t_{0}^{(j)}) \) begin either at the start or at the half of the optical cycle such that \( t_{0} \) is given by

\[ t_{0}^{(j)} = \frac{\pi}{\omega_{j}} \left( \sum_{k=1}^{\infty} m_{k} \right) \mod 2. \]

By increasing the number of the stairs \( N \), we eventually obtain a converged propagation result.

The present approach includes an important modification in respect to Ref. 26. Namely, the original approach cut the pulse to stairs of a constant length, while assuring a continuity of the phase of the oscillating electric field on the interface of every two neighboring steps. Never-the-less the continuity of the phase does not always assure a continuity of the zeroth derivative of the electric field on the interfaces. In the present improved version of the method, the length of the stairs is adjusted to integer numbers of half periods of the given oscillatory electric fields such that discontinuities of the zeroth derivative of the field are completely avoided.

### E. Basis set aspects of the propagations

The simulations have been calculated using the “minimum” and “converged” basis sets of the field-free states, as

\[ P = 0.1 \, \text{meV}, \]

\[ P = 1 \, \text{meV}, \]

\[ P = 5 \, \text{meV}, \]

where the oscillations in each stair are given by \( \sin \omega_{j}(t + t_{0}^{(j)}) \) and begin either at the start or at the half of the optical cycle such that \( t_{0} \) is given by

\[ t_{0}^{(j)} = \frac{\pi}{\omega_{j}} \left( \sum_{k=1}^{\infty} m_{k} \right) \mod 2. \]
they have been defined in Sec. II C, for the known discrepancies between the results obtained in the Floquet calculations for the two basis sets (Tables II–IV). However, the dynamical calculations prove converged even for the “minimum” basis set, see Fig. 5. Similarly, although the stationary calculations indicated a non-negligible influence of the complex scaling parameter on quasi-energies (Tables III and V), no significant difference between the dynamical simulations obtained for $\theta = 0.25$ and $\theta = 0.45$ is found, as demonstrated in Fig. 5 for the “minimum” basis set of the field-free states. We ought to say that the calculations for $\theta = 0.45$ diverge for long-time simulations, which is most probably explicable by positive definite widths of the “transient states,” which occur in the field-free helium spectrum near ionization thresholds. Such states arise as an artifact of the finite ExTG5G Gaussian basis set.26, 27

F. Time-asymmetric phenomena

The final populations (i.e., after the pulse has finished) of the ground and excited states depend on the pulse length $\tau$, as demonstrated in Fig. 5 for the six different chirps based on the
FIG. 6. The eight plots demonstrate changes for different contours (defined by $P$), which are applied either in the counter-clockwise or clockwise sense. The plots show normalized populations of two dynamically coupled Floquet states (solid lines) corresponding to the field-coupled states of helium, $X^1S^+$ and $2^1P^o(a)$ (dashed lines). Initially, only the ground state is populated. The upper panels of the figures illustrate the imaginary part of the energy split between the two Floquet states, which is initially positive but eventually changes its sign. The populations of the Floquet states flip only after the change of the sign of $\text{Im} \Delta E(t)$. The flip of the Floquet states always occurs for the same time (relative to the pulse length $\tau$), supposed that the pulse is sufficiently long—pulses of such lengths are illustrated ($\tau = 150$ fs for (a) and (b), $\tau = 100$ fs for (c) and (d), $\tau = 75$ fs for (e) and (f), and $\tau = 50$ fs for (g) and (h)). The time-delay between the Floquet flip and the sign-change of $\text{Im} \Delta E(t)$ is very short for a contour that tightly encircles the exceptional point (a) and (b); therefore, the Floquet states flip within both negative and positive chirps, which have no excitation effect. As the contours become larger (c) and (f), the Floquet flip is more delayed after the sign-change of $\text{Im} \Delta E(t)$, and thus occurs only partially in the negative chirps, while fully in the positive ones: this leads to the time-asymmetry, where both field-free states are populated after the negative chirps, while the positive chirps have no excitation effect. For yet a larger contour (g) and (h), the Floquet flip does not occur within the pulse length of the negative chirp at all, but it does occur within the positive chirp, which leads to the typical time-asymmetric states exchange phenomenon.
clockwise and counter-clockwise propagation along the three contours $P \in \{0.1; 1; 5\} \, mE_h$. For the most tight contour, $P = 0.1 \, mE_h$, Figs. 5(a) and 5(b), we obtain almost the same pictures for the clockwise and counter-clockwise cases. The final relative populations are exchanged by up to 10% for the short 25 fs pulses, while the longer chirps beyond 200 fs have no excitation effect. However, a time-asymmetric population switch is present for a larger contour, $P = 1 \, mE_h$, Figs. 5(c) and 5(d). An obvious time-asymmetry is present already for the short pulses beyond 10 fs, where the final populations for the clockwise and counter-clockwise pulses are very different. The typical time-asymmetric switch is observed for the pulses longer than 80 fs. It is also worth noting that in the clockwise case, the final populations oscillate with the pulse length in a way reminding of Rabi oscillations (compare Fig. 6(c) in Ref. 26). For the largest contour, $P = 5 \, mE_h$, Figs. 5(e) and 5(f), the time-asymmetry is apparent for the short pulses beyond 2 fs, while the full time-asymmetric population switch would be probably obtained for $\tau > 20$ fs (here, the quantum-dynamical simulations could not be done for the reason of numerical instability).

Clearly, the simulations indicate that the asymmetric switch does not take place for the contour $P = 0.1 \, mE_h$, which encircles the exceptional point very tightly, but it occurs for the larger contours $P = 1$ and $5 \, mE_h$. In order to find an explanation to this phenomenon, we examine the populations of the coupled quasi-energy states during the dynamics for the different contours. The populations are based on the normalized projections of the dynamical electron wavefunction $\phi_{GS}^{(r)}(r; t)$ onto the corresponding time-sections of the quasi-energy states $\phi_{GS}(r; t; \varepsilon_0(t), \omega(t))$, $\phi^{(r)}(r; t; \varepsilon_0(t), \omega(t))$. The normalization compensates for the overall population decay due to ionization and is done similarly to the field-free states, Eq. (16). The quasi-energy states are defined for the instantaneous laser intensity and frequency, $\varepsilon_0(t)$, $\omega(t)$, which correspond to the given time-point on the contour/pulse. Allow us to specify here the notation $+/−$ such that the “+” quasi-energy state corresponds to the helium ground state at $t = 0$, while the “−” quasi-energy state corresponds to the doubly excited resonance state at $t = 0$, such that

$$\Phi^{(r)}(r, t = 0) \equiv \Phi^{(0)}_{GS}(r; t = 0),$$

$$\Phi^{(r)}(r, t = 0) \equiv \Phi^{(0)}_{GS}(r; t = 0).$$

(21)

This definition implies that $\text{Im} \Delta_E(t) = \text{Im}(E_+ - E_-)$ is always positive at the beginning of the chirped pulse, whether it is based on a clockwise or counter-clockwise encircling of the contour.

Populations of the Floquet states in the case of the tight contour, $P = 0.1 \, mE_h$, are illustrated in Figs. 6(a) and 5(b) for both the counter-clockwise and clockwise pulses, respectively. We present populations as functions of relative time $t/\tau$, which are converged in the infinite-pulse limit (here 150 fs). Here, the populations of the Floquet states flip in the course of the pulse both in the counter-clockwise and clockwise pulses. Figures 6(a) and 5(b) indicate that the “+” quasi-energy state starts to be coupled to the “−” quasi-energy state at $t = 0.5 \tau$ and $t = 0.2 \tau$ for the counter-clockwise and clockwise pulses, respectively. Note that the beginning of the population exchange of the two Floquet states is apparent yet before the change of the sign of the imaginary part of the energy split that is depicted above the plots and occurs only at $t = 0.7 \tau$ and $t = 0.35 \tau$ for the counter-clockwise and clockwise pulses, respectively. The fact that the two Floquet states are flipped due to the non-adiabatic coupling in both counter-clockwise and clockwise pulses clues as to why the asymmetric switch of the field-free states does not occur for the tight contours.

Figures 6(c) and 6(d) and Figs. 6(e) and 6(f) show cases of the $P = 0.55 \, mE_h$ and $P = 0.6 \, mE_h$ contours which represent a transient behavior between the tight and large contours, thus giving a clue as to why the atomic switch occurs for larger contours. Again, we study the converged pulse lengths that are now 100 and 75 fs, respectively. These cases differ from the previous one in the fact that the beginning of the flip of the Floquet states is shifted from the early times to $t = 0.7 \tau$ and $t = 0.3 \tau$ for the counter-clockwise and clockwise pulses, respectively. As a consequence, the flip of the quasi-energy states is not complete for the counter-clockwise pulses, leading to a partial switch of the field-free states.

For yet larger contours, the flip of the quasi-energy (Floquet) states does not occur within the counter-clockwise pulses at all, as illustrated for $P = 1 \, mE_h$ in Fig. 6(g), but it remains in the corresponding clockwise pulses, Fig. 6(h). Therefore, we obtain the asymmetric population switch for such cases. The apparently crucial fact that allows for the asymmetric atomic switch is that the flip of the Floquet states does not correspond exactly to the change of the sign of $\text{Im} \Delta_E(t)$ within the contour but may be largely shifted from it so that the pulse is too short to reach the expected flip of the Floquet-states populations.

IV. CONCLUSIONS

When the time-asymmetric switch was originally predicted for the chirps that encircle the exceptional points, it was discussed and proven for the case where the imaginary part of the complex energy split of the two interaction non-Hermitian states does not change its sign in the course of the contour encircling the exceptional point. In such a case, the nonadiabatic coupling, which plays the key role in the photoexcitation precisely due to the imaginary part of the complex energy split, causes that a specific one of the two coupled adiabatic Floquet states prevails in the course of the photoexcitation dynamics.

In our case, the time-asymmetric switch is very sensitive to the shape and size of the closed contour in the laser parameter space. Note that this result is remotely reminiscent of a previous numerical experience of Leclerc, Jolicard, and Killingbeck.35 The present photo-excitation dynamics, such as in other cases studied before, can be ascribed mainly to two interacting Floquet states. The present Floquet Hamiltonian corresponds (in the two-level approximation indeed equals) to the case of the doubly intersecting complex energy surfaces (DICES), which have been known in nuclear dynamics.36 Therefore, here as in the DICES, the exceptional points are paired in the laser parameter space and, even more importantly, a radially diverging seam in the imaginary parts of the two major coupled quasienergies begin at each exceptional
point. Any contour that encircles such an exceptional point cannot but cross the seam. This implies that the quasienergy split of the two major coupled Floquet states that are visited along the contour includes a change of sign in its imaginary part. We show that the change of the sign in the imaginary part of the quasienergy split during the propagation along the contour is a necessary but not sufficient condition for getting the both coupled adiabatic Floquet states subsequently populated in the course of the photoexcitation. This situation typically takes place for the contours that tightly encompass the both coupled adiabatic Floquet states subsequently ing again to the absence of the population switch, while in larger contours, where the two interacting adiabatic Floquet pulses. However, we show a different numerical evidence for the absence of the population switch, although we do not have a clear sufficient condition to populate the two adiabatic states and the change in the imaginary part of the energy split happens in the same stage of the propagation in the small loops as in the large loops. This is shown in Figure 6.

At this point, we have a strong evidence that the change of the sign in the imaginary part of the quasienergy split during the propagation along the contour is a necessary but not sufficient condition to populate the two adiabatic states and then not to have a switch, although we do not have a clear understanding as to why. Even in view of this fact, we can conclude that in this work we provide a realistic example that can be tested by experiments that will show the possibility to obtain a time-asymmetric atomic switch.

ACKNOWLEDGMENTS

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APPENDIX: CALCULATION OF CONTOURS OF A REAL PART OF THE ENERGY SPLIT BETWEEN THE COUPLED FLOQUET STATES

The contour points \([\omega(t; P), \varepsilon_0(t; P)]\) should demonstrate a constant, smooth orbiting motion in the \(\varepsilon_0 - \omega\) plane as the parameter \(t\) linearly changes from 0 to \(\tau\). Therefore first, we define properly scaled coordinates \(\omega_s\), \(\varepsilon_0_s\) taking into account generally different scales of frequency and field strength units such that

\[
\omega_s = \frac{\omega(t; P) - (E_{ES} - E_{GS})/\hbar}{\Delta_{\omega}(P)}, \quad \varepsilon_0_s = \frac{\varepsilon_0(t; P)}{\Delta_{\varepsilon_0}(P)},
\]

(A1)

where \(E_{GS}\) and \(E_{ES}\) are the field-free energies of the helium atom that are coupled by the field, i.e., the ground state energy and the real part of the energy of the doubly excited resonance, respectively; and \(\Delta_{\omega}(P)\) and \(\Delta_{\varepsilon_0}(P)\) are defined as the widths of the contour, \(\Delta_{\omega}(P) = \max \omega(t; P) - \min \omega(t; P), \Delta_{\varepsilon_0}(P) = \max \varepsilon_0(t; P) - \min \varepsilon_0(t; P).\) The scaling factors are determined in an iterative procedure, where \(\Delta_{\omega} = -\text{Re}2\varepsilon_0\) and \(\Delta_{\varepsilon_0} = \text{Re}b/2\varepsilon_0\) are used as a reasonable initial guess for the first iteration, while every iteration means a calculation of the contour points \([\omega(t; P), \varepsilon_0(t; P)]\) using a numerical integration procedure that will be described below. Once the contour points are calculated for the estimated scaling factors, new \(\Delta_{\omega}\) and \(\Delta_{\varepsilon_0}\) are obtained according to their proper definitions, and the procedure is repeated until convergence, where the last obtained contour points \([\omega(t; P), \varepsilon_0(t; P)]\) represent the wanted result.

The numerical integration procedure is based on the second-order-differences algorithm such that

\[
\frac{d\omega_s}{dt} = z_0(t) \left[ 1 + \left( \frac{\partial \varepsilon_0_s}{\partial \omega_s} \right)_P \right]^{-1/2},
\]

(A2)

\[
\frac{d\varepsilon_0_s}{dt} = z_f(t) \left[ 1 + \left( \frac{\partial \varepsilon_0_s}{\partial \varepsilon_s} \right)_P \right]^{-1/2},
\]

(A3)

where \(z_0\) and \(z_f\) are equal to \(\pm 1\) and are determined based on zeros and infinities of the tangents \((\partial \varepsilon_0_s/\partial \omega_s)_P\). The derivative \((\partial \varepsilon_0_s/\partial \omega_s)_P\) is calculated from the partial derivatives of \(P(\omega, \varepsilon_0)\) such that

\[
\left( \frac{\partial \varepsilon_0_s}{\partial \omega_s} \right)_P = \frac{\Delta_{\varepsilon_0}(P)}{\Delta_{\omega}(P)} \left( \frac{\partial P}{\partial \omega_s} \right)_P \frac{\Delta_{\varepsilon_0}(P)}{\Delta_{\omega}(P)} \varepsilon_0_s. \tag{A4}
\]

By substitution from Eq. (14), we get

\[
\left( \frac{\partial \varepsilon_0_s}{\partial \omega_s} \right)_P = \frac{\Delta_{\varepsilon_0}(P)}{\Delta_{\omega}(P)} \left( \frac{\partial P}{\partial \omega_s} \right)_P \frac{\Delta_{\varepsilon_0}(P)}{\Delta_{\omega}(P)} \varepsilon_0_s.
\]

(A5)

where

\[
D_o = a + 2a_2(\omega - \omega^{(EP)}) + c_1(\varepsilon_0 - \varepsilon_0^{(EP)}),
\]

\[
+ c_2(\varepsilon_0 - \varepsilon_0^{(EP)})^2,
\]

\[
D_f = b + 2b_2(\varepsilon_0 - \varepsilon_0^{(EP)}) + c_1(\omega - \omega^{(EP)})
\]

\[
+ 2c_2(\omega - \omega^{(EP)})(\varepsilon_0 - \varepsilon_0^{(EP)}), \tag{A6}
\]

Note that the integration procedure is numerically more stable when using initial conditions defined by \((\partial \varepsilon_0_s/\partial \omega_s)_P = 0\). Note also that \(\omega(t; P)\) and \(\varepsilon_0(t; P)\) orbit around the exceptional point in the counter-clockwise direction. When the clockwise direction is to be examined, the pulses are simply reversed such that \(t/\tau \rightarrow (1 - t/\tau)\).